# THE CUBIC FORMULA AND ITS CONSEQUENCES

A Math Teachers' Circle Report

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Available at `https://mkjanssen.org/talks.html`

# INTRO

- Story of the cubic formula is one of my favorites in mathematics
- Spoke on it in a department colloquium in the fall
- One of the identities we explored raised a question about complex numbers that I though it would be fun to explore in a math teachers circle setting
- My colleague, Tom Clark, runs a math teachers' circle for area teachers (also attended by math education majors)
- Idea for the session: tell the story of the discovery of the cubic, and explore related mathematical ideas
- MTC: low floor, high ceiling, guided discovery

# STARTING THE ACTIVITY

- Exercise: Depressed cubics are enough
- Exposition: 1535: Fiore vs Tartaglia, and one of Fiore's problems
- Exposition: Cardano and Tartaglia, Tartaglia's poetry

# DERIVING THE FORMULA

Exercise: Starting with a depressed cubic $u^3 + eu + f = 0$, rewrite

$$(s + t)^3 = s^3 + 3s^2t + 3st^2 + t^3$$

to see that $u = s + t$ is a solution if $e = -3st$ and $f = -(s^3 + t^3)$.

Exercise: solve for $s$ and $t$ in terms of $e$ and $f$ to obtain the formula for a root.

# A DELIGHTFUL IDENTITY

Exercise. Solve $x^3 - 15x - 4 = 0$ to obtain

$$x = \sqrt[3]{2 + 11i} + \sqrt[3]{2 - 11i} = 4.$$

# COMPLEX NUMBERS

- Previous session covered the basics of arithmetic in $\mathbb{C}$
- How to identify $\sqrt[3]{2 + 11i} = 2 + i$:
  - Exercise. Algebra: verify $a = 2$, $b = -1$ is a solution to $(a + bi)^3 = 2 + 11i$
  - Exercise. Trig via the polar form
- Exercise. Roots of unity

# RECEPTION

- It went well!
- Some surprises

## THE END

# Thanks for coming!