THE SURPRISINGLY SPICY STORY OF THE CUBIC FORMULA

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September 6, 2024

Some (Pre)History

Linear Equations: The Babylonians

Problem (Al-Khwārizmī, *The Condensed Book on the Calculation of al-Jabr and al-Muqabala* (825))

What must be the square which, when increased by ten of its roots, amounts to thirty-nine?

Solution: The solution is this: you halve the number of roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this which is eight, and subtract from it half the number of the roots, which is five; the remainder is three. This is the root of the square which you sought for.

KHAYYAM'S CLASSIFICATION OF CUBICS

Binomial:

$x^3 = d$

Trinomial:

$x^3 + cx = d$

$x^3 + d = cx$

$x^3 = cx + d$

$x^3 + bx^2 = d$

$x^3 + d = bx^2$

$x^3 = bx^2 + d$

Tetranomial:

$x^3 + bx^2 + cx = d$

$x^3 + bx^2 + d = cx$

$x^3 + cx + d = bx^2$

$x^3 = bx^2 + cx + d$

$x^3 + bx^2 = cx + d$

$x^3 + cx = bx^2 + d$

$x^3 + d = bx^2 + cx$

The Italians

CONTEXT

Known as late as 1494 that one could reduce a general cubic

$$ax^3 + bx^2 + cx + d = 0$$

to the so-called depressed cubic

$$y^3 + py + q = 0$$

So, solving the depressed cubic would yield a general solution.

MODERN VS RENAISSANCE ACADEMIA

- In Renaissance Italy there was a dependence on rich patrons, and positions were temporary
- One needed to continually prove oneself worthy, generally through public competitions
- Thus, if you made a discovery, you'd typically hold it back for use against your opponent.
- Today, you are rewarded for publicizing your discoveries

1535: FIORE VS TARTAGLIA

- Scipione del Ferro (1465–1526) was a professor at the University of Bologna who discovered how to solve cubics of the form $x^3 + cx = d$.
- Antonio Maria Fiore was a student of del Ferro, who passed the solution on to him.
- Niccolò Fontana (1499–1557, aka *Tartaglia*) discovered how to solve $x^3 + bx^2 = d$.
- Fiore heard of Tartaglia's claim and challenged him to a competition in 1535.

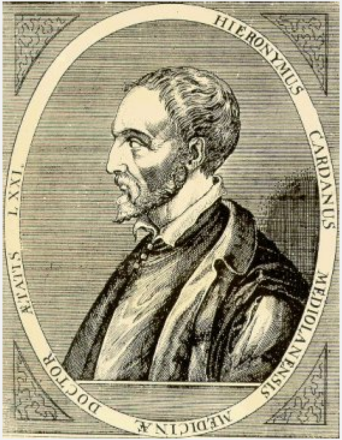
Problem

A man sells a sapphire for 500 ducats, making a proft of the cube root of his capital. How much is this profit?

- Tartaglia worked to figure out how to solve cubics of that form; Fiore couldn't figure out Tartaglia's cubics and lost the competition.

GEROLAMO CARDANO (1501–1576)

- Public lecturer in mathematics
- Wrote to Tartaglia, wanting to include his solution to the cubic in a new arithmetic text
- In 1539, Tartaglia relented and came to Milan
- Cardano pledged an oath not to publish Tartaglia's solution
- Tartaglia gave the solution in a poem



TARTAGLIA'S POEM

For $x^3 + cx = d$:

When the cube and its things near
Add to a new number, discrete,
Determine two new numbers different
By that one; this feat
Will be kept as a rule
Their product always equal, the same,
To the cube of a third
Of the number of things named.
Then, generally speaking,
The remaining amount
Of the cube roots of subtracted
Will be your desired count.

THE ARS MAGNA

- Cardano kept his promise not to publish Tartaglia’s result in the new arithmetic book
- Cardano began to work on the problem himself, assisted by his student Lodovico Ferrari (1522–1565)
- Worked out all the cases in the coming years; Tartaglia still hadn’t published

Cardano’s Formula

OBVIOUSLY

$$\sqrt[3]{\sqrt{-121}+2}+\sqrt[3]{-\sqrt{-121}+2}=4$$

CODA

- Cardano heard a rumor that the original solution had been found by del Ferro in Bologna, so he went to see del Ferro's son-in-law, who showed Cardano del Ferro's notes
- Cardano no longer felt an obligation to Tartaglia; instead he'd publish *del Ferro's* solution, discovered 20 years earlier
- 1545: the *Ars Magna* is published, and includes solutions to the cubic and quartic, and acknowledged Tartaglia's work

Questions?